Rossmoyne Senior High School

WA Exams Practice Paper D, 2015

Question/Answer Booklet

MATHEMATICS METHODS UNIT 1
Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

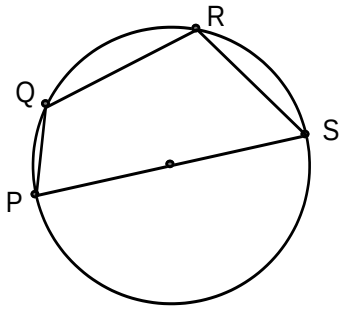
Section Two: Calculator-assumed **(98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 9 **(7 marks)**

P, Q, R and S are points that touch the circumference of a circle with diameter 22 cm. PS is a diameter of the circle, PQ has length 6 cm and RS has length 11 cm.



(a) Determine the area of the minor segment bound by the chord RS. (2 marks)

$$A = \frac{1}{2}(11)^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)$$
$$= 10.961 \text{ cm}^2$$

(b) Determine the length of the minor arc PQ. (3 marks)

cosine rule to find angle QOP:

$$\theta = \cos^{-1}\left(\frac{11^2 + 11^2 - 6^2}{2(11)(11)}\right)$$
$$\theta = 0.55245^r$$
$$L = r\theta$$
$$= 11(0.55245)$$
$$= 6.077 \text{ cm}$$

(c) Find the area of quadrilateral PQRS. (2 marks)

$$A_1 = \frac{1}{2} \times 11 \times 11 \times \sin(\pi/3)$$
$$A_2 = \frac{1}{2} \times 11 \times 11 \times \sin(0.55245)$$
$$A_3 = \frac{1}{2} \times 11 \times 11 \times \sin(\pi - \pi/3 - 0.55245)$$
$$A = 52.396 + 31.749 + 60.475 = 144.62 \text{ cm}^2$$

See next page

Question 10 **(12 marks)**

(a) Let set A be defined as $A = \{ x : x$ is a positive integer less than 10 $\}$.

(i) State $n(A)$. (1 mark)

9

(ii) Explain whether the statement $0 \in A$ is true or false. (1 mark)

False. Zero is not a positive integer and so is not an element of set A.

(b) Let set $B = \{ 1, 2, 3, 4 \}$. An integer is formed by choosing two digits, at random and without repetition, from B.

(i) List the sample space in a systematic way. (1 mark)

{12, 13, 14, 21, 23, 24, 31, 32, 34, 41, 42, 43 }

(ii) Determine the probability that the integer formed is a prime number. (1 mark)

$\frac{5}{12}$

(iii) Determine the probability that the integer formed is even, given that it is a multiple of three. (1 mark)

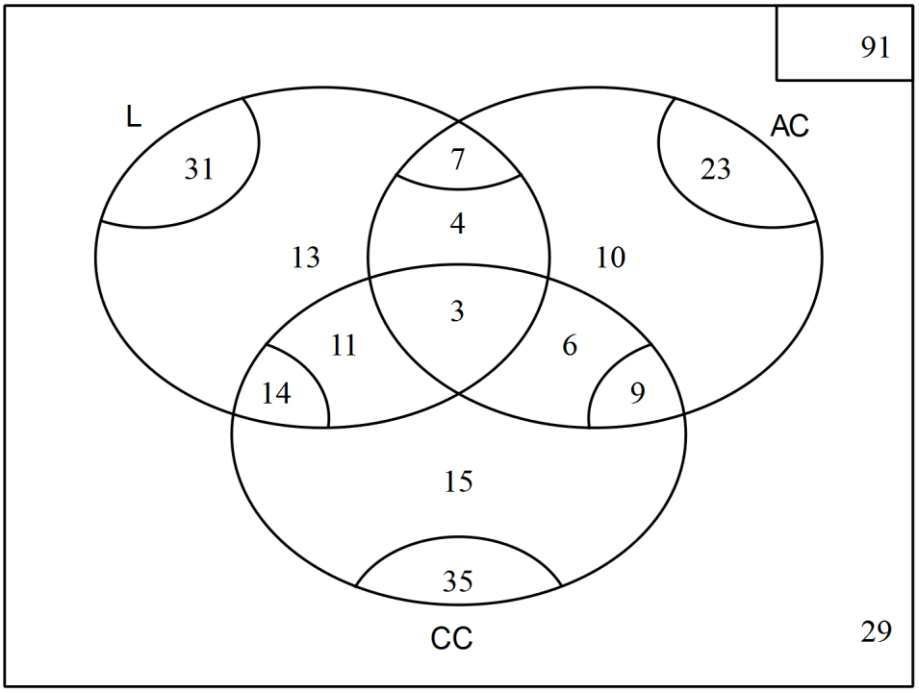
$\frac{3}{4}$

(c) Determine the number of ways in which a shortlist of 5 photographs can be chosen from a choice of 20 different photographs. (2 marks)

$^{20}C_5 = 15504$

(d) Following a survey of the use of three council facilities it was found that 31 people had used the library and 23 had used the aquatic centre. Of the 7 who had used both the aquatic centre and the library, 3 had also used the community centre. 10 people had only used the aquatic centre whilst 11 people had used the library and the community centre but not the aquatic centre. 29 of the 91 surveyed had used none of the facilities.

(i) Complete the Venn diagram below to show this information, showing the number of people in each region of the diagram. (3 marks)



(ii) If one person surveyed was selected at random, what is the probability that they used no more than two facilities if it was known that they had used at least one of the facilities? (1 mark)

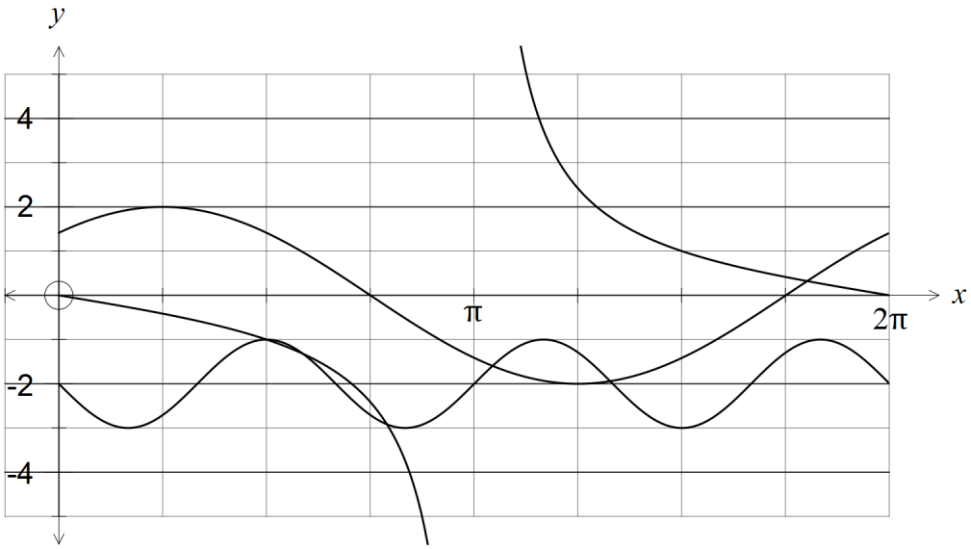
$$\frac{91-29-3}{91-29}=\frac{59}{62}$$

(iii) Explain why using the library and using the aquatic centre are not mutually exclusive events. (1 mark)

The intersection of these two sets is not empty.

Question 11 **(6 marks)**

The graphs of the functions $y = a\cos(x + b)$, $y = \sin(cx) + d$, $y = e\tan(fx)$ are shown below, where a, b, c, d, e and f are real constants.



State the values of constants a, b, c, d, e and f .

$a = 2$

$b = -\frac{\pi}{4}$

$c = -3$

$d = -2$

$e = -1$

$f = 0.5$

(alternative: $e = 1$, $f = -0.5$)

Question 12 **(6 marks)**

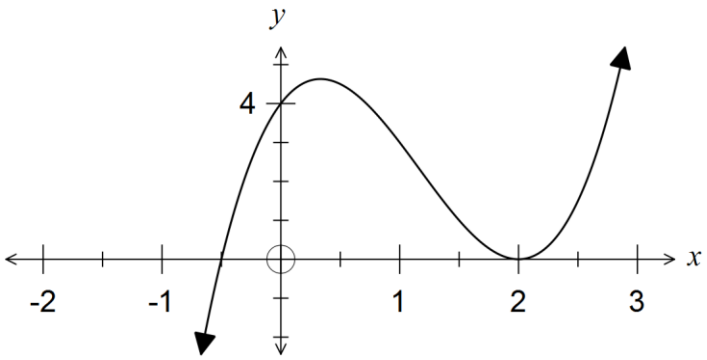
(a) The graph of $y = x^2 + px + q$ has a turning point at (3, -2). Determine the coordinates of the y-intercept of the parabola. (2 marks)

$y = (x-3)^2 - 2$

$y = (-3)^2 - 2$
$\quad = 7$

$(0,\ 7)$

(b) The graph of $y = ax^3 + bx^2 + cx + d$ is shown below.



Determine the values of $a,\ b,\ c$ and d. (4 marks)

$y = a(x+0.5)(x-2)^2$

$4 = a(0.5)(4) \Rightarrow a = 2$

$y = 2(x+0.5)(x-2)^2$
$\quad = 2x^3 - 7x^2 + 4x + 4$

$a = 2,\ b = -7,\ c = 4,\ d = 4$

See next page

Question 13 **(8 marks)**

The table below summarises the use of various health services on at least one occasion by people from different household structures over a 12 month period.

Household structure	All persons		No use	GP	Specialist	Dentist	Hospital	Other
Person living alone	2 330		151	2 074	994	1 056	379	583
Couple only	4 834		226	4 372	2 155	2 475	724	1 284
Couple with children	10 198		915	8 259	2 716	4 887	998	1 908
All other households	4 744		512	3 824	1 242	1 925	498	769
Total	22 106		1 804	18 529	7 107	10 343	2 599	4 544

For example, out of the 22 106 people surveyed, 10 198 lived in a household structured as a couple with children, and of these 10 198 people, 2 716 had used the services of a health care specialist on at least one occasion over the 12 month period.

(a) Of those people who lived in a household structure with either a couple only or a couple with children, what percentage had used the services of a GP? (2 marks)

$$\frac{4372+8259}{4834+10198}\times 100=\frac{12631}{15032}\times 100$$

$$=84.0\%$$

(b) If one person is selected at random from those surveyed, determine the probability that

(i) they had made use of 'other' health services? (1 mark)

$$\frac{4544}{22106}$$

$$\left(\approx 0.206\right)$$

(ii) they had used the services of a specialist and their household structure was 'all other'. (1 mark)

$$\frac{1242}{22106}$$

$$\left(\approx 0.056\right)$$

See next page

(iii) they had made no use of health services, given that they lived alone. (1 mark)

$$\frac{151}{2330}$$

$$(\approx 0.065)$$

A researcher was investigating whether use of hospital services was independent of type of household structure.

(c) Determine the probability that a randomly selected person used hospital services

(i) given that they lived alone. (1 mark)

$$\frac{379}{2330} \approx 0.163$$

(ii) given that they lived as a couple with children. (1 mark)

$$\frac{998}{10198} \approx 0.098$$

(d) Is use of hospital services independent of type of household structure? Justify your answer. (1 mark)

No, not independent, as using figures in (c), probability of using hospital services is about 1.5 times greater for persons living at home than for couples with children.

Question 14 **(7 marks)**

(a) A parabola with vertex at (2, -1) passes through the points (3, 0) and (3, -2).

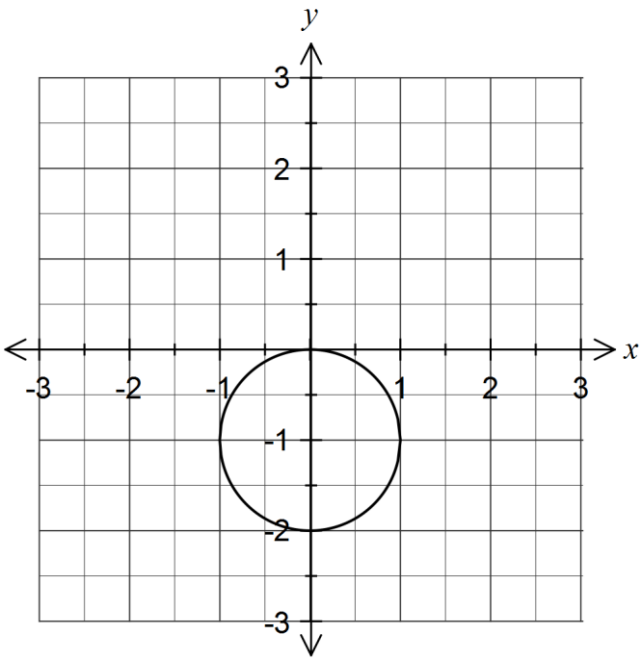
(i) State the equation of the axis of symmetry for this parabola. (1 mark)

$y = -1$

(ii) Determine the equation of the parabola. (2 marks)

$(y+1)^2 = x-2$

(b) Sketch the graph of $x^2 + (y+1)^2 = 1$. (2 marks)



(c) Show that the circle with equation $x^2 + y^2 - 22x + 30y + 310 = 0$ has a radius of 6. (2 marks)

$(x-11)^2 + (y+15)^2 = -310 + 11^2 + 15^2$

$(x-11)^2 + (y+15)^2 = 36$

$r = 6$

See next page

Question 15 **(6 marks)**

(a) Determine the smallest positive solution to the equation $3\cos(3x)=1$, where x is in degrees, giving your solution to three significant figures. (2 marks)

23.5°

(b) Show that $\cos(2\theta)=\cos^2\theta-\sin^2\theta$. (2 marks)

$$\begin{aligned}\cos(2\theta)&=\cos(\theta+\theta)\\&=\cos\theta\times\cos\theta-\sin\theta\times\sin\theta\\&=\cos^2\theta-\sin^2\theta\end{aligned}$$

(c) Show that the exact value of $\tan 105°$ is $-\left(\sqrt{3}+2\right)$. (2 marks)

$$\begin{aligned}\tan(105)&=\tan(45+60)\\&=\frac{\tan(45)+\tan(60)}{1-\tan(45)\tan(60)}\\&=\frac{1+\sqrt{3}}{1-\sqrt{3}}\\&=-\sqrt{3}-2\\&=-\left(\sqrt{3}+2\right)\end{aligned}$$

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Question 16 **(7 marks)**

In triangle ABC, $a = 15.4$ cm, $b = 12.8$ cm and $\angle B = 50°$.

(a) If ABC is an **acute-angled** triangle,

(i) write down an equation that could be solved to determine the size of $\angle A$. (1 mark)

$$\frac{15.4}{\sin A} = \frac{12.8}{\sin 50}$$

(ii) determine the size of $\angle A$. (1 mark)

$$A = 67.2°$$

(b) If ABC is an **obtuse-angled** triangle,

(i) write down an equation that could be solved to determine the length c. (2 marks)

$$\angle A = 180 - 67.2 = 112.8° \Rightarrow \angle C = 180 - 112.8 - 50 = 17.2°$$

$$\frac{c}{\sin 17.2} = \frac{12.8}{\sin 50} \quad \text{or} \quad 12.8^2 = c^2 + 15.4^2 - 2c(15.4)\cos 50$$

(ii) determine the length c. (1 mark)

$$c = 4.93 \text{ cm}$$

(c) Determine the largest possible area of triangle ABC. (2 marks)

Largest if triangle ABC is acute angled.

$$\angle C = 180 - 67.2 - 50 = 62.8°$$

$$\text{Area} = 0.5(15.4)(12.8)\sin 62.8$$
$$= 87.7 \text{ cm}^2$$

See next page

Question 17 **(7 marks)**

A factory uses three machines to produce one type of plastic bottle. Of the total production, machine A produces 35%, machine B produces 25% and machine C the rest. Due to the age of the machines, they all produce some defective bottles. Of their production, machines A and B produce 3% and 6% defective bottles respectively.

(a) Find the probability that a randomly selected bottle is produced by machine A and is defective. (1 mark)

$$\begin{aligned} P(A \cap D) &= P(A) \times P(D \mid A) \\ &= 0.35 \times 0.03 \\ &= 0.0105 \end{aligned}$$

(b) If the probability of a randomly selected bottle being defective is 0.0455, what percentage of the production of machine C is defective? (4 marks)

Machine C has 100% - 35% - 25% = 40% of production.

$P(A \cap D) = 0.0105$

$P(B \cap D) = 0.25 \times 0.06 = 0.015$

$P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D)$

$0.0455 = 0.0105 + 0.015 + P(C \cap D)$

$\therefore P(C \cap D) = 0.02$

$P(C \cap D) = P(C) \times P(D \mid C)$

$0.02 = 0.4 \times P(D \mid C)$

$\therefore P(D \mid C) = 0.05$

5% of output from C is defective.

(Tree diagram or 2-way table are alternative methods here)

(c) Given that a randomly selected bottle is not defective, find the probability that it was produced by either machine A or machine B. (2 marks)

$$\begin{aligned} &\frac{P(A \cap \bar{D}) + P(B \cap \bar{D})}{1 - 0.0455} \\ &= \frac{0.35 \times 0.97 + 0.25 \times 0.94}{0.9545} \\ &= 0.6019 \end{aligned}$$

See next page

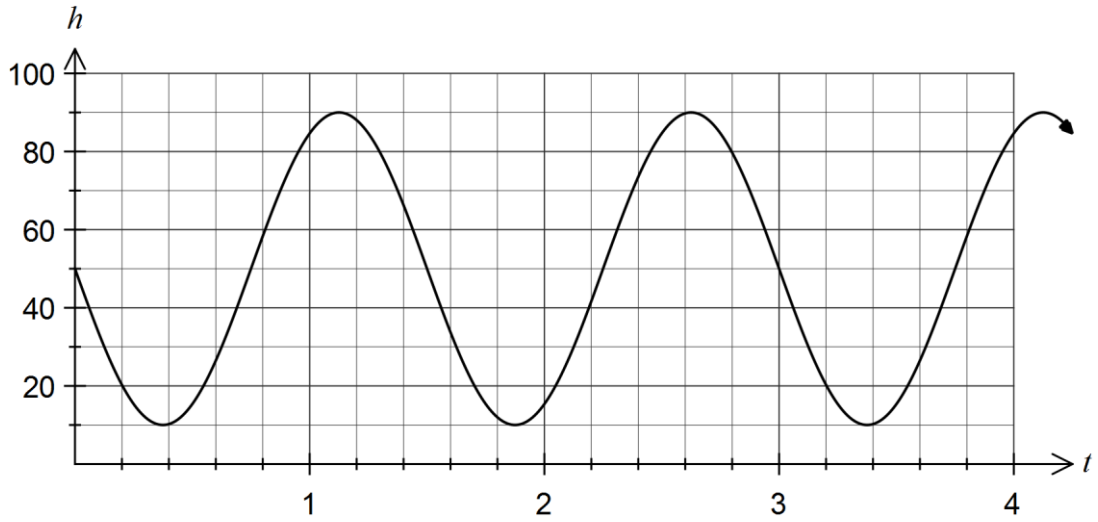
Question 18 **(9 marks)**

The height above the ground, h cm, of a small weight attached to a spring was observed to vary according to the function $h(t) = 50 - 40\sin\left(\frac{4\pi t}{3}\right)$, where t is the time in seconds since measurements began.

(a) Determine the initial height of the weight above the ground. (1 mark)

50 cm

(b) Sketch the graph of $h(t)$ on the axes below for $0 \le t \le 4$. (3 marks)



(c) What is the period of the motion? (1 mark)

$2\pi \div \frac{4\pi}{3} = 1.5$ seconds

(d) How far did the weight travel during the first six seconds? (2 marks)

Travels 4 complete cycles in 6 s.
Travels 160 cm each cycle.

Travels $4 \times 160 = 640$ cm

(e) For how long during the first six seconds was the weight within 20 cm of the ground? (2 marks)

Solve $20 = 50 - 40\sin\left(\frac{4\pi t}{3}\right)$ to get solutions of $t = 0.202$ and $t = 0.548$.

$0.548 - 0.202 = 0.346$ in 1.5 s, so $4 \times 0.346 = 1.38$ seconds in 6 s.

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Question 19 **(8 marks)**

For events A and B, $P(A) = a$, $P(B) = b$ and $P(\bar{A} \cap \bar{B}) = 0.1$.

(a) Determine an expression for $P(A \cap B)$ in terms of a and b. (2 marks)

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= a + b - (1 - 0.1) \\ &= a + b - 0.9 \end{aligned}$$

It is also known that $P(A \mid B) = 0.4$.

(b) Determine an expression for a in terms of b. (2 marks)

$$P(A \mid B) = \frac{a + b - 0.9}{b}$$

$$\frac{a + b - 0.9}{b} = 0.4$$

$$a = 0.9 - 0.6b$$

(c) Determine the values of a and b under each of the following conditions.

(i) $P(A \cap \bar{B}) = 0.3$. (2 marks)

$$\begin{aligned} P(A \cap \bar{B}) &= a - (a + b - 0.9) \\ 0.3 &= 0.9 - b \\ b &= 0.6 \end{aligned}$$

$$\begin{aligned} a &= 0.9 - 0.6(0.6) \\ a &= 0.54 \end{aligned}$$

(ii) A and B are independent. (2 marks)

$$P(A) = P(A \mid B)$$

$$a = 0.4$$

$$0.4 = 0.9 - 0.6b$$

$$b = \tfrac{5}{6}$$

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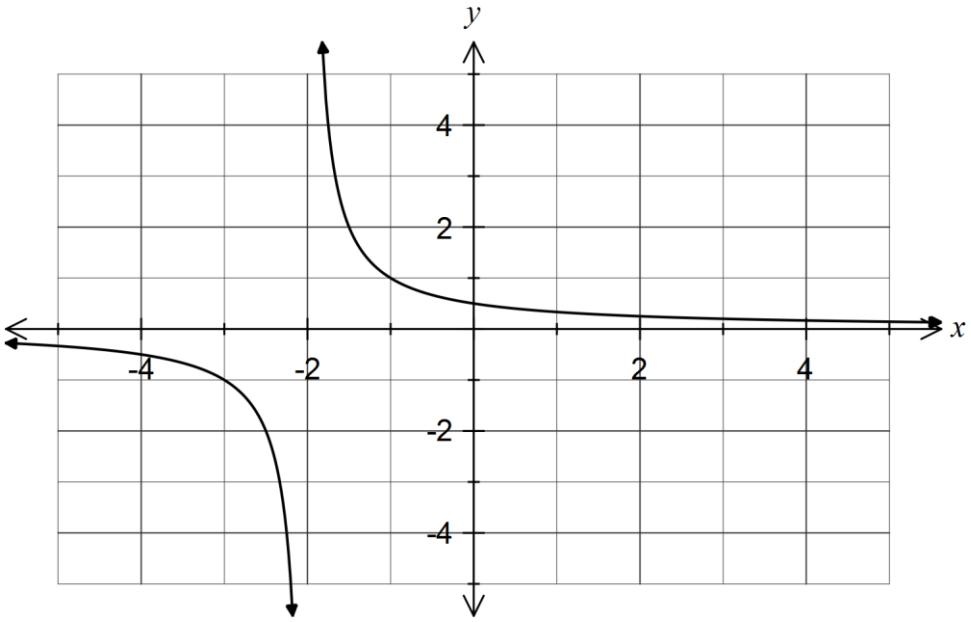
Question 20 **(8 marks)**

The function f is defined by $f(x)=\dfrac{1}{x+2}$.

(a) State the domain and range of $f(x)$. (2 marks)

$\{x: x\in\mathbb{R},\ x\neq -2\}$ and $\{y: y\in\mathbb{R},\ y\neq 0\}$

(b) Sketch the graph of $y=f(x)$. (2 marks)



(c) State the equations of the asymptotes of the graph of $y=f(x)+3$. (2 marks)

$x=-2$ and $y=3$

(d) Describe how to transform the graph of $y=f(x)$ to obtain the graph of

(i) $y=f(4x)$. (1 mark)

Dilate the graph horizontally by a scale factor of $\frac{1}{4}$.

(ii) $y=\dfrac{1}{x-1}$. (1 mark)

Translate the graph 3 units to the right.

See next page

Question 21 **(7 marks)**

(a) Use an algebraic method to determine the point of intersection of $y = \frac{2}{x-1}$ and $y = \frac{1}{2x+1}$. (3 marks)

$$\frac{2}{x-1} = \frac{1}{2x+1}$$
$$4x + 2 = x - 1$$
$$3x = -3$$
$$x = -1$$
$$y = \frac{2}{-1-1} = -1$$

At the point $(-1, \ -1)$

(b) The quantity P is directly proportional to another quantity h and when $h = 3.5$, $P = 1225$.

Determine the value of h when $P = 7000$. (2 marks)

$$\frac{h}{7000} = \frac{3.5}{1225}$$
$$h = 20$$

(c) The quantity C is inversely proportional to t. It is known that $C = 30$ when $t = 24$.

Determine the value of C when $t = 7.2$. (2 marks)

$$30 \times 24 = C \times 7.2$$
$$C = 100$$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

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